Teaching Spatial Visualization via Spherical Mechanism Design

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Abstract

Our effort to integrate spatial geometry into an undergraduate mechanisms course has focused upon incorporating finite position synthesis of spherical $4R$ mechanisms, whose synthesis equations closely resemble those of the planar $4R$ mechanism, into the curriculum to teach spatial visualization. In the course students use *Sphinx*, an interactive graphics design package for designing spherical mechanisms developed at U.C. Irvine. Our experience has been that by using the software to design spherical mechanisms, students gain the visual insight needed to comprehend general spatial motion.

1 Introduction

This paper discusses our efforts to integrate spatial geometry into an undergraduate mechanism design course. For four years we have worked to incorporate finite position synthesis of spherical $4R$ mechanisms into a standard course on $4R$ planar mechanism analysis and design. Spherical $4R$ mechanisms have four rigid bodies connected by revolute joints whose axes intersect at a point, in contrast to planar mechanisms which have axes that are parallel. For more information regarding spherical mechanisms see Chiang, 1988, and Duffy, 1980. The result of our efforts has been the development and student use of, *Sphinx*, an interactive graphics package for designing spherical mechanisms. In the course, students gain the visual insight needed to grasp and comprehend the concept of rotational motion.

We use a linear algebra formulation so that, mathematically, planar and spherical finite position synthesis look the same. However, a major difference arises due to the essential three dimensionality of spherical mechanisms. Our experience has been that the leap from the $2D$ graphical constructions of planar mechanism design to $3D$ space is not difficult conceptually, however, in space the constructions can not physically be performed. Therefore the students benefit from two visualization tools we have developed: *Sphinx* and the spherical mechanism prototyping kit. In using the software, students gain the visual insight needed to grasp and comprehend the concept of spherical motion, thereby enabling them to successfully design spherical mechanisms. Experience has demonstrated that the knowledge gained by students while using *Sphinx* provides them with the ability to construct simple, inexpensive, and functional spherical $4R$ mechanisms using the spherical mechanism prototyping kit. Using the kit, students develop their intuition and manufacturing skills for spherical mechanism construction.

Spherical mechanisms generate new and interesting movements that are three dimensional by nature. Moreover, since general spatial motion can be studied as a a combination of linear and rotational motions, once spherical mechanisms have been understood, the student is well prepared for an introduction to the study of spatial geometry; especially robotics, computational geometry, and geometric modeling.

2 Spatial Displacements

We now present the equation for the general spatial displacement of a point in terms of the translation and rotation of its reference frame with respect to the frame's initial position, see Bottema and Roth,

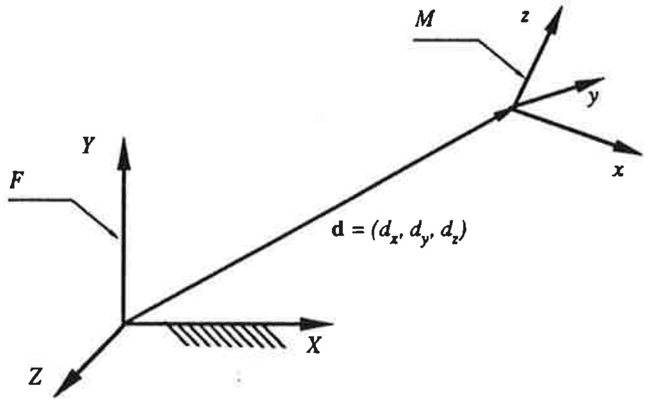


Figure 1: A General Spatial Displacement

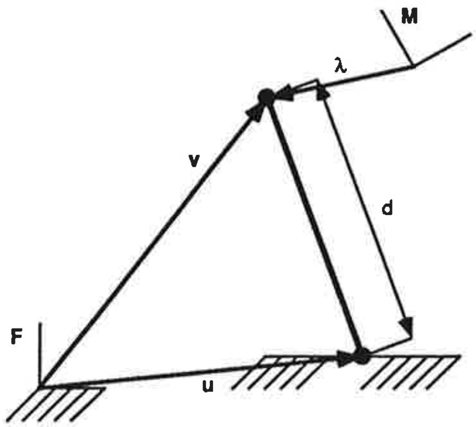


Figure 2: A Planar RR Dyad

1979, and McCarthy, 1990. The coordinates of a general point $\mathbf{x}$, measured in the moving frame M, initially coincident with a frame F (which may be the fixed reference frame), which undergoes a general spatial displacement satisfy, see Fig. 1,

$$\mathbf{X} = [A]\mathbf{x} + \mathbf{d} \qquad (1)$$

where, $[A]$ is the 3×3 orthonormal rotation matrix representing the orientation of frame M relative to frame F and $\mathbf{d} = [d_x, d_y, d_z]^T$ is the translation vector from the origin of frame F to the origin of frame M.

For most students a general spatial motion is a difficult concept to grasp. By decomposing spatial displacements into terms involving pure translations, $\mathbf{d}$, and pure rotations, $[A]$, we simplify the concept of spatial motion. In general students can visualize a spatial translation. However, our experience has been that spatial rotations pose a greater challenge to students. Since all spatial rotations can be viewed as motion on the surface of a sphere and because the links of a spherical mechanism are constrained to move on the surface of a sphere, spherical mechanism design serves as an ideal tool for teaching the visualization of spatial rotations.

3 Planar $4R$ Synthesis

A planar RR dyad is shown in Fig. 2. Let the axis of the fixed joint be specified by the vector, $\mathbf{u}$, measured in the fixed reference frame and let the moving axis be specified by, $\boldsymbol{\lambda}$, measured in the moving frame, M. Let us define the vector $\mathbf{v}$ as representing the moving axis $\boldsymbol{\lambda}$ as measured in the fixed frame F. The vectors $\mathbf{v}$ and $\boldsymbol{\lambda}$ are related by, $\mathbf{v} = [A]\boldsymbol{\lambda} + \mathbf{d}$. Because the two axes are connected by a rigid link the distance between the two axes of the dyad remains constant. This rigid body geometric constraint is written in equation form as,

$$\begin{aligned} (\mathbf{v}-\mathbf{u})\cdot(\mathbf{v}-\mathbf{u}) &= d^2 \\ ([A]\boldsymbol{\lambda}+\mathbf{d}-\mathbf{u})\cdot([A]\boldsymbol{\lambda}-\mathbf{d}-\mathbf{u}) &= d^2 \qquad (2) \end{aligned}$$

The methodology for performing the dimensional synthesis of planar $4R$ mechanisms for three position rigid body guidance is as follows, see Sandor and Erdman, 1991, and Suh and Radcliffe, 1978. This algorithm is based upon synthesizing two dyads separately and then joining them with a coupler to form a planar $4R$ closed chain mechanism. First, we select the two moving pivots $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$. Second, we write Eq. 2 for each of the desired positions, $([A], \mathbf{d})_i, i = 1, 2, 3$. Then, we subtract the first equation from the remaining two to arrive at a linear system of equations,

$$[P]\mathbf{u} = \mathbf{b} \qquad (3)$$

where,

$$[P] = \begin{bmatrix} 2(\mathbf{v}_2^T - \mathbf{v}_1^T) \\ 2(\mathbf{v}_3^T - \mathbf{v}_1^T) \end{bmatrix}$$

$$\mathbf{b} = \begin{pmatrix} \mathbf{v}_2^T\mathbf{v}_2 - \mathbf{v}_1^T\mathbf{v}_1 \\ \mathbf{v}_3^T\mathbf{v}_3 - \mathbf{v}_1^T\mathbf{v}_1 \end{pmatrix}$$

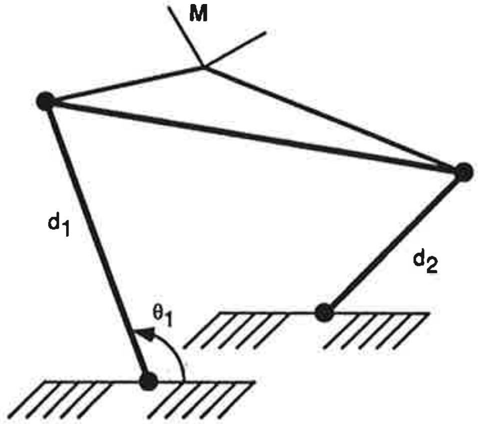


Figure 3: A Planar 4*R* Mechanism

$\mathbf{v}_i$ are the coordinates of the moving pivot in the i^{th} position, and finally $\mathbf{u}$ is the desired fixed pivot. Note that we must solve Eq. 3 for each desired moving pivot to find its corresponding fixed pivot. A complete planar 4*R* mechanism is shown in Fig. 3.

4 Spherical 4*R* Synthesis

A spherical *RR* dyad is shown in Fig. 4. Let the axis of the fixed joint be specified by the vector, $\mathbf{u}$, measured in the fixed reference frame and let the moving axis be specified by, $\boldsymbol{\lambda}$, measured in the moving frame, M. Because the two axes are connected by a rigid link the angle between the two lines of the dyad remains constant. This rigid body geometric constraint is written in equation form as,

$$\begin{aligned} \mathbf{v}\cdot\mathbf{u} &= \cos\alpha \\ [A]\,\boldsymbol{\lambda}\cdot\mathbf{u} &= \cos\alpha \qquad (4) \end{aligned}$$

The methodology for the dimensional synthesis of spherical 4*R* mechanisms for three position rigid body guidance follows the same procedure as outlined above for planar mechanisms. Performing the appropriate steps we arrive at,

$$[P]\mathbf{u} = \mathbf{b} \qquad (5)$$

where,

$$[P] = \begin{bmatrix} \boldsymbol{\lambda}^T([A]_2 - [A]_1)^T \\ \boldsymbol{\lambda}^T([A]_3 - [A]_1)^T \\ 0 \quad 0 \qquad 1 \end{bmatrix}$$

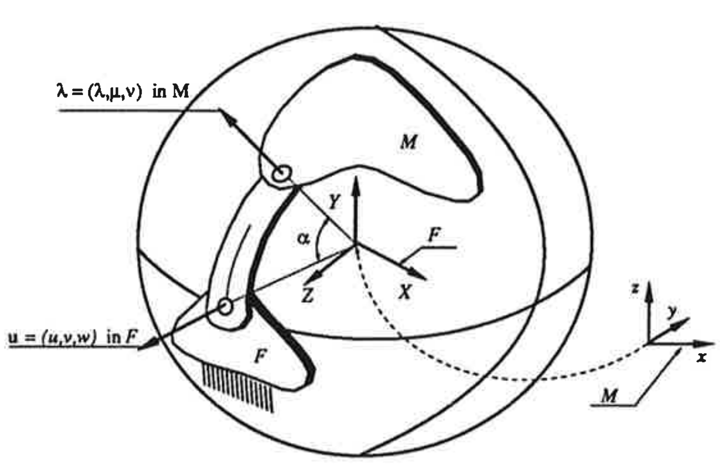


Figure 4: A Spherical *RR* Dyad

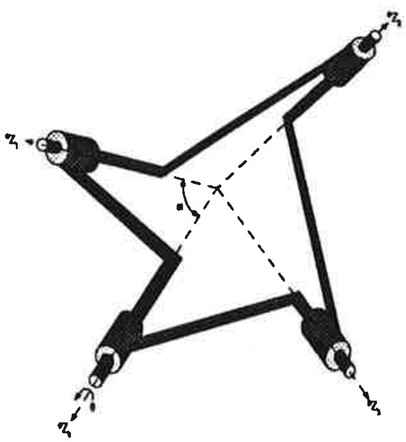


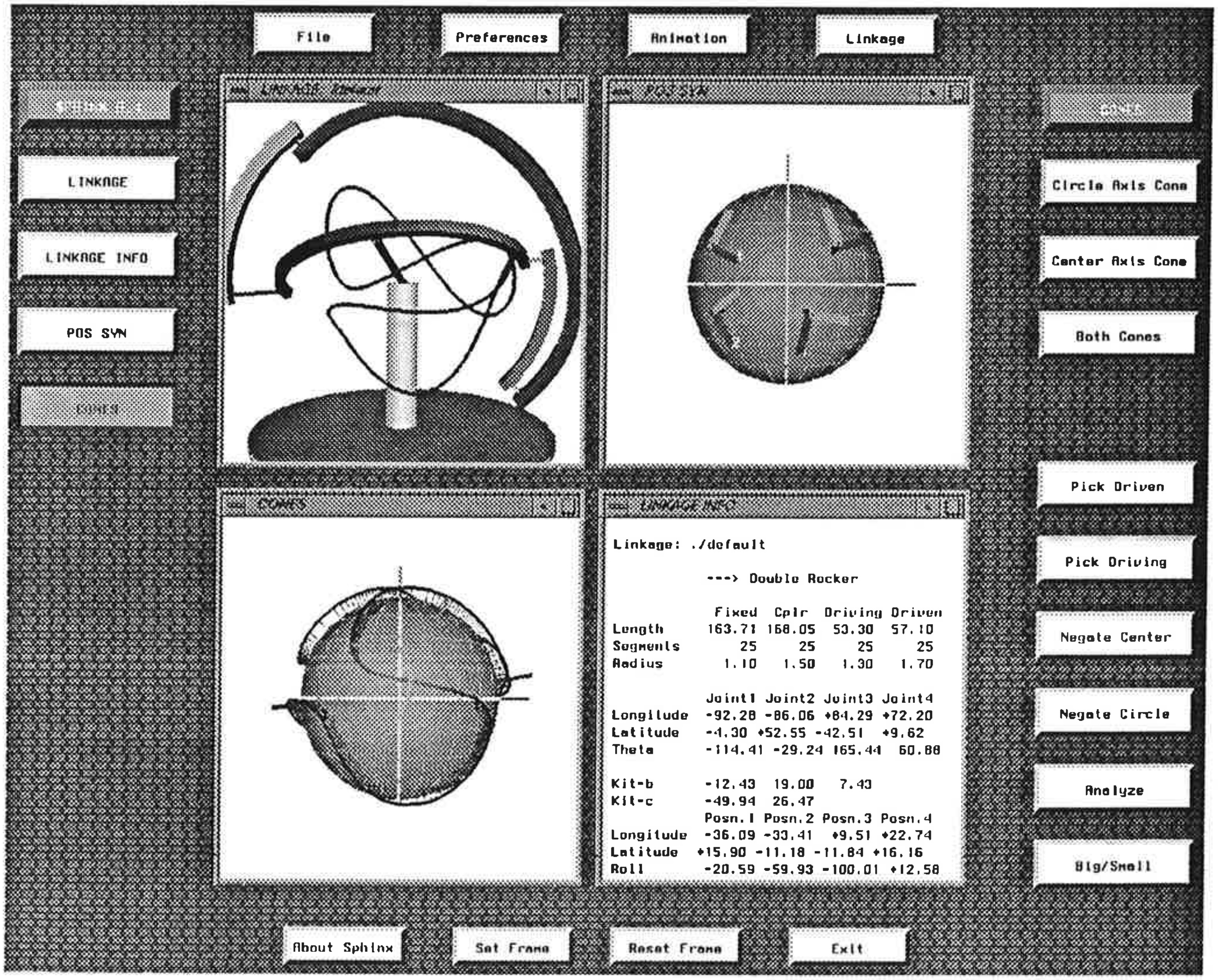
Figure 5: A Spherical 4*R* Mechanism

$$\mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

and $\mathbf{u}$ is the desired fixed pivot. Note that we must solve Eq. 5 for each desired moving pivot to find its corresponding fixed pivot. A complete spherical 4*R* mechanism is shown in Fig. 5.

5 *S*PHINX

*S*PHINX is a computer graphics based interactive program for designing spherical mechanisms formed by a closed chain consisting of four revolute joints; the so called spherical 4*R* mechanism. For further discussion of *S*PHINX, see Larochelle et al, 1993. The



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Linkage: ./default

           ---> Double Rocker

            Fixed   Cplr  Driving Driven
Length     163.71 168.05   53.30   57.10
Segments       25     25      25      25
Radius       1.10   1.50    1.30    1.70

           Joint1 Joint2 Joint3 Joint4
Longitude  -92.28 -86.06 +84.29 +72.20
Latitude   -4.30 +52.55 -42.51  +9.62
Theta      -114.41 -29.24 165.44  60.88

Kit-b      -12.43  19.00   7.43
Kit-c      -49.94  26.47
           Posn.1 Posn.2 Posn.3 Posn.4
Longitude  -36.09 -33.41  +9.51 +22.74
Latitude  +15.90 -11.18 -11.84 +16.16
Roll       -20.59 -59.93 -100.01 +12.58
```

Figure 6: *S*PHINX

result of the design is a one degree of freedom mechanism which guides a body through finitely separated orientations in space. In SPHINX the user specifies orientations, using longitude, latitude, and roll angles, which are displayed as positions on the surface of a sphere. This display is generated by translating the reference frame of the orientation from the origin of the sphere along its z axis a unit distance so that the reference frame appears on the surface of a unit sphere, see Fig. 6. Our experience has been that this technique enhances the student's ability to visualize spatial orientations. If the translation were not done all of the orientations would have coinciding origins at the center of the sphere thereby making visualization difficult.

The major difference between planar and spherical finite position synthesis is the essential three dimensionality of spherical mechanisms. While synthesis constructions for planar mechanisms may be sketched or plotted in two dimensions, those for spherical mechanisms must be viewed in their full three dimensional form. Moreover, in the spherical case, the mechanism itself requires a three dimensional representation. We have found that static visualization is insufficient, and that in order to successfully design spherical mechanisms the user requires the ability to interactively manipulate the sphere and view it in an arbitrary orientation. By using the graphics capabilities of a Silicon Graphics 4D-85 GT, SPHINX provides the three dimensional interactive environment needed to design spherical $4R$ mechanisms.

6 The Prototyping Kit

We have designed, developed, and manufactured a spherical mechanism prototyping kit(patent pending) which can be used independently, or in conjunction with SPHINX. The purpose of the kit is to aid students in the construction of an initial prototype of their design. By physically constructing their designs students gain an appreciation of machines and mechanisms that can provided spatial and spherical motions. Moreover, they develop basic manufacturing skills for spherical mechanism construction. To aid in assembling the kit SPHINX provides the user with a three dimensional visualization of their mechanism design as it should be assembled with the spherical mechanism prototyping kit, see Fig. 6.

7 Conclusion

Our experience has been that teaching spatial displacement theory in the context of spherical mechanism design with SPHINX and the spherical mechanism prototyping kit has: *i* provided a methodology for students to understand, and develop an intuition for, spatial motion. *ii* sparked interest and enthusiasm in students in that they are designing and building things they've never seen before, *iii* resulted in students that are well prepared for an introduction to more involved studies of spatial geometry; including robotics, computational geometry, and geometric modeling.

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